Advertised Prices in Decentralized Markets

Derek Stacey*

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Abstract

A model of a decentralized market is developed that features search frictions, advertised prices and bargaining. Sellers can post ask prices to attract buyers through a process of directed search, but *ex post* there is the possibility of renegotiation. Similarly, buyers can advertise negotiable bid prices to attract sellers. Even though transaction prices often differ from quoted prices, advertised bid and ask prices play a crucial role in directing search and reducing trading frictions. The features and predictions of the model align well with aspects of the secondary market for transferable taxicab license plates in Toronto. This provides a useful and unique context for studying the relationships between advertised and actual prices in a decentralized market.

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*Ryerson University, Department of Economics, 350 Victoria Street, Toronto, Ontario, Canada, M5B 2K3, `dstacey@economics.ryerson.ca`. Helpful comments were received from participants of the Vienna Macro Conference (2014), the Housing-Urban-Labor-Macro (HULM) Conference (Spring 2015), the Society for Economic Dynamics (SED) Annual Meeting in Warsaw (2015), and seminars at McMaster and Wilfrid Laurier.

1 Introduction

In this paper I develop a dynamic model of a decentralized market that features search frictions, advertised bid and ask prices, and transaction prices determined by bargaining. Buyers and sellers have the capacity to post advertisements to attract a trading partner through a process of directed search. The advertisement communicates commitment to a bid or ask price, but the counterparty that has not engaged in pre-match price publication maintains the ability to trigger *ex post* renegotiation. In a setting where there is *ex ante* uncertainty about a trader's relative strength at the bargaining table, bid and ask prices are chosen strategically as a means of directing search. The theory provides insight about how advertised prices and transaction prices are determined in equilibrium when there is limited commitment by only one party to an advertised price. The possibility of unfavorable outcomes in the bargaining procedure justifies the prevalence of posted prices even if advertised bid and ask prices tend to be different from transaction prices.

The model characterizes an asset market with trading frictions. The trading process has three features: pre-match communication, search frictions, and a strategic method of price determination. The model assumes that traders on either side of the market have the opportunity to post a public advertisement. Many decentralized markets feature some form of public medium for advertising market participation and attracting trading partners by including a *bid price* (a buyer's offer-to-purchase price) or an *ask price* (a seller's quoted price). Accordingly, the communication stage of the trading process is modeled as publicly advertised bid and ask prices. Next, buyers and sellers meet stochastically according to a matching technology. This feature reflects the presence of search frictions; it is necessary to first find and contact a trading partner before a transaction can take place which results in trading delays. The model assumes that matches occur between traders that advertise a price and those that instead decide to actively search. Transaction prices are determined by bargaining between a matched buyer and seller, where the advertised price is interpreted as the initial offer in an alternating offer bargaining game. In a setting where there is *ex ante* uncertainty about a trader's relative bargaining strength, negotiated premiums/discounts relative to advertised bid/ask prices arise in equilibrium whenever bargaining favors the trading partner that is not constrained by a commitment to an advertised price.

Search is *directed* in the sense that searching traders can observe all price announcements and target a particular advertised price in their search. In such settings, posted prices can provide incentives for potential trading partners to direct their search towards them. If the seller's expected bargaining strength is sufficiently high, for example, an ask price effectively limits the seller's share of the surplus and can therefore be chosen strategically as a means of attracting a buyer. The strategic role of an ask price is therefore somewhat related to that in Chen and Rosenthal (1996a,b) and Arnold (1999), where a seller sets an asking price to effect a price ceiling which encourages a buyer to incur the cost of inspecting the item for sale.[1] In a setting where sellers compete for buyers, Lester, Visschers, and Wolthoff (2014) show that an asking price mechanism provides both an appropriate means of attracting buyers and sufficient motivation to incur the inspection cost. Even in the absence of idiosyncratic values (observable or otherwise) and costly inspection, I show that an appropriately chosen ask price should appeal to buyers if it insures them against unfavorable outcomes in price negotiations. Moreover, an advertised bid price permits a buyer to implement an analogous technique for seducing sellers.

The model is a hybrid of random and directed search models. In the literature

[1] An asking price is relevant in these settings because buyers have idiosyncratic valuations (*i.e.*, willingness to pay) that become common knowledge after the inspection. Consequently, buyers might capture too little of the expected surplus to justify incurring the inspection cost unless sellers commit ahead of time to a price ceiling.

on random search, the terms of trade are typically determined *ex post* by means of a bargaining protocol (Diamond, 1982; Mortensen, 1982a,b; Pissarides, 1984, 1985; Mortensen and Pissarides, 1994). In contrast, the terms of trade are publicly posted in a competitive search model akin to those studied by Montgomery (1991), Peters (1991), and Moen (1997), and market participants have the ability to commit not to renegotiate. In many markets, the actual transaction prices often differ from the advertised terms of trade because of *ex post* negotiation. The search model proposed in this paper can account for this and at the same time establishes a link between an advertised price and the trader's matching probability. Despite traders' limited ability to use a bid or ask price as a firm commitment to transact at a particular price,[2] the practice of advertising prices results in expected transaction prices that satisfy the Hosios (1990) condition in equilibrium. Negotiable advertised prices thus efficiently reduce trading frictions despite an otherwise generically inefficient *ex post* bargaining procedure. A novel and appealing part of the theory relative to the existing directed search literature is that the set of traders advertising prices is not imposed exogenously. Rather, the decision to advertise a bid or ask price is determined in equilibrium and depends on the details of the bargaining procedure.[3] The potential for both buyers and sellers to advertise prices is crucial for generic constrained efficiency.

In equilibrium, ask prices exceed the average transaction price, whereas bid prices lie below the average sale price. These predictions differ dramatically from directed search models that adopt a reserve price interpretation of the advertised price, as in Peters and Severinov (1997) and Julien, Kennes, and King (2000), or models that

[2]Other papers in the directed search literature have studied the efficiency implications of removing or relaxing the assumption of commitment to a posted price (*e.g.*, Menzio (2007); Kim and Kircher (2014); Stacey (2014); and Albrecht, Gautier, and Vroman (2015)). In this paper, a trader can commit to honor the advertised price, but their counterparty in a match may not agree to it and instead elect to negotiate.

[3]Halko, Kultti, and Virrankoski (2008) also endogenize search direction (*i.e.*, the side of the market making offers), but do not model publicly advertised offers. They focus on residual wage dispersion when wages are determined in sealed-bid auctions.

study limited commitment to an initial offer that is not contingent on realized demand, as in Albrecht, Gautier, and Vroman (2015) and the strategic renegotiation process in Camera and Selcuk (2009). The search models presented in these papers are appropriate for studying markets that are sufficiently active or unbalanced that multilateral matches are common. For example, houses in high-demand neighborhoods often sell above the list price when multiple buyers compete by submitting offers to purchase the same unit. In contrast, the model developed in this paper best describes less active markets where it is unlikely that a buyer or seller will be in contact with more than one potential trading partner at the same time.[4]

Search and bargaining models currently represent a prominent theoretical framework for studying over-the-counter (OTC) financial markets in which investors must identify a suitable counterparty and interact bilaterally in order to carry out a transaction. Models of unmediated decentralized asset markets with random search and bargaining include, among others, Duffie, Gârleanu, and Pedersen (2007); Hugonnier, Lester, and Weill (2014); and Afonso and Lagos (2015). The competitive search equilibrium approach with full commitment to posted prices is briefly introduced in the context of unmediated OTC markets by Rocheteau and Weill (2011), while Watanabe (2013) and Lester, Rocheteau, and Weill (2014) apply competitive search to asset markets where trades are intermediated by dealers. In contrast, I formalize a model of decentralized exchange that features both directed search and bargaining. For applications to specific OTC financial markets, the environment is sufficiently tractable that it could be extended relatively easily along several dimensions to include, for example, trader heterogeneity, private information, and intermediation.

[4]The model could be extended to account for transactions both above and below advertised prices by embellishing the matching technology to include occasional multilateral meetings. For example, in the rare event that several buyers match with a single seller, *ex post* competition among buyers could result in a transaction price above the seller's advertised ask price. The role of advertised prices would remain the same as long as multilateral meetings are sufficiently unlikely.

As an interesting application of the model, I consider the secondary market for transferable standard taxicab licenses (STLs) in Toronto. These are license plates that can not only be used by an owner-driver to operate a taxicab vehicle in Toronto, but can also be leased, rented to shift drivers or transferred to a new owner by means of a transaction in the secondary market. This market provides a unique context that is particularly appropriate for examining the implications of search frictions and the role of advertised prices because the assets being exchanged (*i.e.*, the standard taxicab licenses) are homogeneous, but are nevertheless traded in a decentralized manner; buyers and sellers publish prices and search for each other by means of an online platform for classified advertisements. The advertised prices and transaction data support the theoretical implications insofar as advertised ask prices are typically higher than transaction prices, while advertised bid prices are lower. The model parameters are calibrated to match certain features of the STL market, and the equilibrium of a stochastic version of the dynamic model exhibits advertised and transaction price distributions that are similar to those observed in the data. These similarities lend support the mechanism proposed in the theory based on strategically chosen bid and ask prices that direct search but are subject to bilateral negotiation.

The remainder of the paper is organized as follows. Section 2 describes the model environment. The equilibrium strategies for bargaining and price posting are characterized in Section 3. Section 3 also defines a steady state directed search equilibrium with advertised prices. Section 4 provides an overview of the secondary market for STLs and describes the recent transaction data as well as the data on advertised prices. Section 5 presents the calibrated model and discusses the role of bid and ask prices in directing search and the incidence of renegotiation. Section 6 concludes.

2 The Environment

Time is discrete and indexed by t. There is a large number of infinitely-lived agents. Some of them are initially endowed with one indivisible long-lived asset. Let A denote the number of agents endowed with the asset, or equivalently, the total supply assets in the economy. For simplicity, assume that each agent can hold at most one unit of the asset.[5] The asset yields a dividend d each period.

Preferences. Agents are risk-neutral with common discount factor $\beta \in (0, 1)$. An agent that owns an asset is subject to a random preference shock that can reduce his valuation of the dividend from d to $(1 - x)d$, with $x > 0$. Conditional on holding an asset, the preference shock arrives with probability δ each period. Once an agent experiences this shock, his valuation of the dividends from his particular asset will remain low forever. This captures the idea that some agents that own an asset might develop a need for selling it, which generates churning in the market.

Trading process. Ownership of the asset can be transferred in a decentralized market subject to search frictions described below. Once the asset is sold, the seller exits the market with the revenue from the sale. Traders therefore transition to a different trading status depending on their asset holdings and their valuation of the asset. There are three different stages that occur sequentially: (i) buyers (n) do not own an asset; (ii) owners (m) have the asset and value its stream of dividends; and (iii) sellers (s) have the asset, but have experienced the preference shock and no longer fully value its dividends. Denote the measure of traders of different types at time t as n_t, m_t, and s_t.

Let $\{V_t^n, V_t^m, V_t^s\}$ denote the expected present discounted values associated with buying, owning, and selling the asset at time t. These values represent the solution to

[5] This simplifies the bargaining problem and avoids having to solve for a more complicated distribution of asset holdings in equilibrium.

the system of Bellman equations derived below. The net surplus from a transaction at time t at price P is therefore $V_t^m - V_t^n - P$ for a buyer, and $P - V_t^s$ for a seller.

Search and matching. The meeting process between buyers and sellers is subject to frictions, but search can be *directed* by advertised prices in the following sense: buyers have the opportunity to advertise a bid price, whereas sellers can post an ask price. Traders that do not post prices then observe all advertised prices and search for a trading partner with a particular bid or ask price. Let p denote the advertised price, and let $s_t(a,p)$ and $n_t(a,p)$ (or $n_t(b,p)$ and $s_t(b,p)$) indicate the numbers of traders posting and searching for ask price (or bid price) p. The first argument indicates whether the advertised price is a seller's ask price or a bid price quoted by a buyer.

Buyers and sellers are matched according to a bilateral meeting technology given by a matching function, $\mathcal{M}(n,s)$. The function $\mathcal{M}$ exhibits constant returns to scale and satisfies the condition that the number of matches is less than the number of traders on the short side of the market, $\mathcal{M}(n,s) \leq \min\{n,s\}$. It is convenient to define $\theta_t(i,p) \equiv n_t(i,p)/s_t(i,p)$ as the ratio of buyers to sellers participating in submarket $(i,p) \in \{a,b\} \times \mathbb{R}_+$, which is referred to as *market tightness*. Given the properties of $\mathcal{M}$, the probability that a buyer will meet a seller in period t may be written as a function of $\theta_t(i,p)$:

$$\lambda(\theta_t(i,p)) = \frac{\mathcal{M}(n_t(i,p), s_t(i,p))}{n_t(i,p)} = \mathcal{M}(1, 1/\theta_t(i,p)). \tag{1}$$

Let $\gamma(\theta_t(i,p))$ denote the analogous matching probability from the perspective of a seller:

$$\gamma(\theta_t(i,p)) = \frac{\mathcal{M}(n_t(i,p), s_t(i,p))}{s_t(i,p)} = \mathcal{M}(\theta_t(i,p), 1) = \theta_t(i,p)\lambda(\theta_t(i,p)). \tag{2}$$

Traders direct their search by posting or targeting a particular p knowing the matching probabilities, given the matching function and their beliefs about the price posting strategies and search behavior of other traders.

Assumption 1. *The function* $\gamma : [0, \infty] \rightarrow [0, 1]$ *is twice continuously differentiable, increasing and concave. Functions* γ *and* λ *satisfy the boundary conditions* $\gamma(0) = \lambda(\infty) = 0$ *and* $\gamma(\infty) = \lambda(0) = 1$.

Price determination It is assumed that buyers and sellers have the capacity to commit to honoring their quoted price. A price poster's counterparty, on the other hand, has not publicly announced any such commitment and therefore reserves the right to renegotiate. Absent agreement to transact at the advertised price, the transaction price of the asset is determined by means of an alternating offer bargaining game. Following Binmore, Rubinstein, and Wolinsky (1986), the buyer and seller bargain strategically when there is a perceived risk that the bargaining process will terminate in disagreement between offers. The trader rejecting the posted price is the first to make a counteroffer.

For example, suppose the seller has advertised an ask price of p which the buyer refuses to pay. In the first round of the renegotiation game, the buyer offers p_1 which the seller can either accept or reject. If the offer is accepted, the bargaining game ends and ownership of the asset is transferred to the buyer in exchange for payment p_1. If p_1 is rejected, negotiations breakdown with probability $1 - \exp(-(1 - \phi)\Delta)$ and both parties continue searching/waiting for a trading partner the following period. If bargaining continues, the seller proposes a price p_2 to be accepted or rejected by the buyer. If rejected, negotiations can again be terminated, this time with probability $1 - \exp(-\phi\Delta)$. The bargaining game continues indefinitely until either negotiations are aborted or a mutually agreeable offer is proposed and accepted. Figure 1 displays the timing of the alternating offer bargaining game. Figure 2 displays the renegotiation process when a seller contacts a buyer that advertised a bid price.

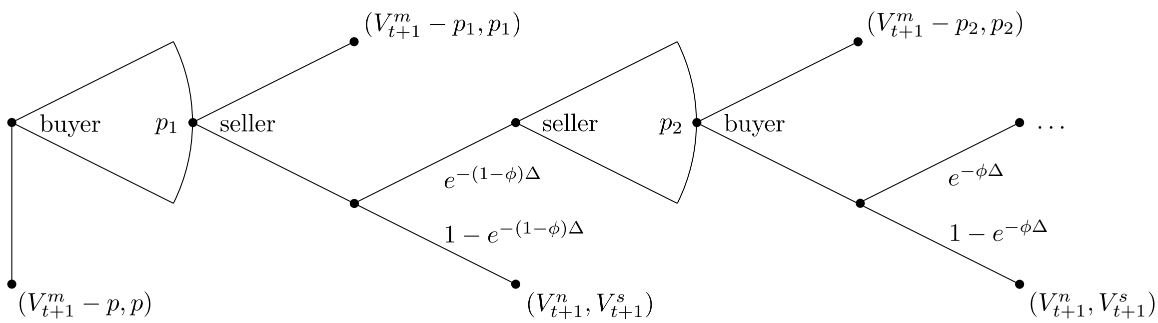

Figure 1: Price determination when the seller is contacted by a buyer.

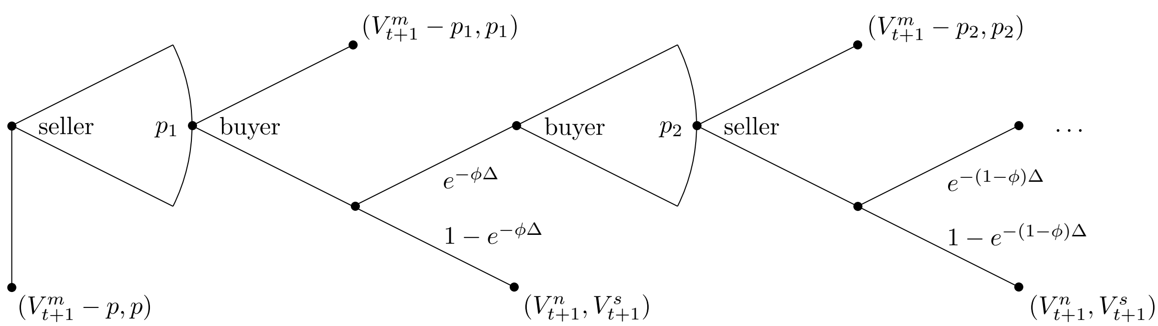

Figure 2: Price determination when the buyer is contacted by a seller.

Consider the limiting subgame perfect equilibrium outcome by letting the length of the bargaining rounds, $\phi\Delta$ and $(1-\phi)\Delta$, decrease to zero for a given $\phi \in [0, 1]$, as in Binmore (1980) and Binmore, Rubinstein, and Wolinsky (1986). This allows the dynamic strategic model of bargaining to be collapsed into a single time period of the dynamic search model. Here, the perceived risk of breakdown can be thought of as a proxy for commitment; the higher (lower) is ϕ, the higher (lower) is the seller's (buyer's) commitment to his offer as the *final* offer, even if rejected. By taking the limit as $\Delta \to 0$, what is important is the bargainer's relative power of commitment, even though neither trader can terminate the bargaining process in disagreement with positive probability.[6]

Search and bargaining models of frictional markets typically assume that the bargaining strength of the seller relative to that of the buyer is constant across all matched buyer-seller pairs. More generally, there might be pre-match uncertainty about traders' tenacity at the bargaining table in terms of their capacity to commit to offers. In other words, an individual's bargaining prowess likely varies over time and may depend on the idiosyncratic characteristics of the bargaining opponent.[7] To incorporate these ideas in a simple and tractable manner, let ϕ be a match-specific random variable. This uncertainty establishes a potential role for an advertised price as a commitment to an initial or preemptive offer in the bargaining game. The counterparty in the match can accept the advertised price, but retains the right to negotiate depending upon the realization of ϕ. If actively searching traders are strategic about when to negotiate, then a passive trader could conceivably advertise a price strategically to attract a potential trading partner.

Assumption 2. *The match-specific bargaining parameter ϕ is a continuous random variable with support $[0, 1]$ and distribution function F.*

Free entry. Participation in the market requires forgoing other potential investment opportunities. An exogenous per period opportunity cost is denoted $c \in (0, d)$. It is worthwhile to enter the market as a buyer at time t as long as the expected present discounted value associated with searching to buy an asset is positive.

[6] This interpretation follows Schelling's (1956) views on strategic bargaining and the model based on commitment in Section 8.7 of Myerson (1991).

[7] For example, Harding, Rosenthal, and Sirmans (2003) provide evidence that buyer and seller attributes influence bargaining power in house price negotiations.

3 Equilibrium

Bargaining outcome. The following lemma combines Propositions 3 and 5 in Binmore, Rubinstein, and Wolinsky (1986). It states that in the absence of agreement to transact at the advertised price, the unique subgame perfect equilibrium outcome of the bargaining game converges to the asymmetric generalization of the Nash solution. The net surplus captured by the seller is a fixed share of the total surplus from a transaction, where the surplus splitting rule is determined by parameter ϕ (*i.e.*, the seller's relative bargaining strength).

Lemma 1. *If the buyer rejects the advertised ask price and engages the seller in the bargaining game, there exists a unique subgame perfect equilibrium. The existence and uniqueness results similarly hold if the seller rejects the advertised bid price and engages the buyer in the bargaining game. In the limit as $\Delta \to 0$, the equilibrium outcome at time t in both cases is*

$$\hat{p}_t = V^s_{t+1} + \phi \left(V^m_{t+1} - V^n_{t+1} - V^s_{t+1} \right). \tag{3}$$

The pre-match expected bargaining outcome at time t in the absence of an acceptable posted price is therefore

$$\mathrm{E}[\hat{p}_t] = V^s_{t+1} + \mathrm{E}[\phi] \left(V^m_{t+1} - V^n_{t+1} - V^s_{t+1} \right), \tag{4}$$

where $\mathrm{E}[\phi] = \int_0^1 \phi \, dF(\phi)$ is the expected value of the bargaining parameter.

Optimal renegotiation. Upon contacting a seller with advertised ask price p, the buyer's optimal strategy is to bargain if $\hat{p}_t < p$ and to accept p otherwise. When a seller targets a buyer with advertised bid price p, the seller elects to renegotiate in order to receive $\max\{p, \hat{p}_t\}$. These optimal strategies are stated in the following lemma using the solution to the bargaining problem in (3).

Lemma 2. *An ask price of p is accepted by the buyer whenever*

$$\phi \geq \frac{p - V_{t+1}^s}{V_{t+1}^m - V_{t+1}^n - V_{t+1}^s} \equiv \Phi_t(p), \tag{5}$$

whereas a bid price of p is accepted whenever $\phi \leq \Phi_t(p)$.

Let $P_t(i, p)$ denote the expected transaction price in a buyer-seller match when the advertised price is p, where $i \in \{a, b\}$ indicates whether p is an ask price posted by the seller or a bid price posted by a buyer. Expected transaction prices are computed taking into account the advertised price, the decision about when to initiate *ex post* negotiations (Lemma 2), and the anticipated outcome of the bargaining game (Lemma 1), given the distribution F of the random bargaining parameter, ϕ. When a seller posts ask price p, the expected transaction price is $P_t(a, p) = \mathrm{E}\left[\min\{p, \hat{p}_t\}\right]$, or

$$P_t(a, p) = \min\left\{p, V_{t+1}^s\right\} + \left(V_{t+1}^m - V_{t+1}^n - V_{t+1}^s\right) \int_0^{\max\{0, \Phi_t(p)\}} \left[1 - F(\phi)\right] \, d\phi. \tag{6}$$

When a seller targets a buyer with advertised bid price p, the expected price conditional on a match is $P_t(b, p) = \mathrm{E}\left[\max\{p, \hat{p}_t\}\right]$, or

$$P_t(b, p) = \max\left\{p, V_{t+1}^s\right\} + \left(V_{t+1}^m - V_{t+1}^n - V_{t+1}^s\right) \int_{\max\{0, \Phi_t(p)\}}^1 \left[1 - F(\phi)\right] \, d\phi. \tag{7}$$

The expected transaction price, given by $P_t(i, p)$ according to (6) and (7), is depicted graphically in Figure 3 to illustrate some important properties summarized in the following lemma.

Lemma 3. *(i) The functions $p \mapsto P_t(a, p)$ and $p \mapsto P_t(b, p)$ are continuous on $\mathbb{R}_+$; (ii) $p \mapsto P_t(a, p)$ is increasing in p on $[0, V_{t+1}^m - V_{t+1}^n)$ and constant when $p \geq V_{t+1}^m - V_{t+1}^n$,* *with* $P_t(a,0)=0$, $P_t(a,V^s_{t+1})=V^s_{t+1}$ *and* $P_t(a,V^m_{t+1}-V^n_{t+1})=\mathrm{E}[\hat{p}_t]$*; (iii)* $p\mapsto P_t(b,p)$ *is constant when* $p\le V^s_{t+1}$ *and increasing in* p *when* $p>V^s_{t+1}$*, with* $P_t(b,V^s_{t+1})=\mathrm{E}[\hat{p}_t]$ *and* $P_t(b,V^m_{t+1}-V^n_{t+1})=V^m_{t+1}-V^n_{t+1}$*; and (iv) the function* $P_t:\{a,b\}\times\mathbb{R}_+\to\mathbb{R}_+$ *is surjective.*

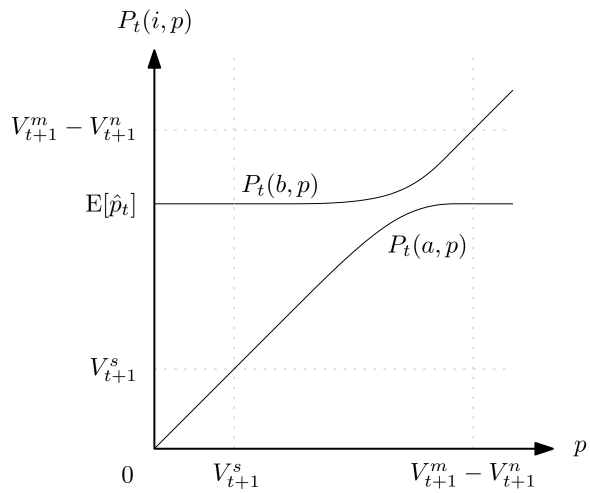

Figure 3: Expected transaction price as a function of the advertised price.

Advertised prices, search and free entry. To study the search decisions and price posting problems of buyers and sellers, it is necessary to first derive the Bellman equations for V^m_t, V^n_t and V^s_t. The present discounted value of owning the asset at time t is given by

$$V^m_t=d-c+\beta\left[(1-\delta)V^m_{t+1}+\delta V^s_{t+1}\right]. \tag{8}$$

Equation (8) states that the value of owning the asset is equal to the current dividend, d, less the opportunity cost, c, plus the expected present discounted value next period; this will either be the value of maintaining ownership, V^m_{t+1}, which occurs with probability $(1-\delta)$, or the value after the preference shock, which occurs with probability δ. In this case, the trader captures the present discounted value of selling the asset, V_{t+1}^s. The values associated with attempts to sell or buy an asset at time t satisfy

$$V_t^s = (1-x)d - c + \beta \max_{i\in\{a,b\},p} \left\{\gamma(\theta_t(i,p))P_t(i,p) + (1-\gamma(\theta_t(i,p)))V_{t+1}^s\right\} \tag{9}$$

$$V_t^n = -c + \beta \max_{i\in\{a,b\},p} \left\{\lambda(\theta_t(i,p))\left(V_{t+1}^m - P_t(i,p)\right) + (1-\lambda(\theta_t(i,p)))V_{t+1}^n\right\}, \tag{10}$$

where maximizing with respect to $i \in \{a,b\}$ reflects the optimal choice between price posting and actively searching. Maximizing with respect to p in (9), for example, reflects the optimal choice of ask price for a seller if $i = a$, and the optimal search decision among buyers with advertised bid prices if $i = b$. In (10) it reflects the optimal choice of bid price or the search decision of a buyer. When there is a match between a buyer and seller, the actual transaction price depends on the subsequent bargaining procedure given the realization of ϕ and the advertised price. Traders understand *ex ante* that the expected payment, $P_t(i,p)$, is consistent with (6) and (7); the final price of an asset depends on all possible equilibrium outcomes of the bargaining game as well as the the optimal renegotiation strategies given the posted price (Lemmas 1 and 2).

Definition 1. *Given $\beta V_{t+1}^n < c < \beta\left(V_{t+1}^m - V_{t+1}^n - V_{t+1}^s\right)$, a directed search equilibrium at time t is a set of ask prices $\mathbb{P}_t^a \subset \mathbb{R}_+$; a set of bid prices $\mathbb{P}_t^b \subset \mathbb{R}_+$; a function for market tightness $\theta_t : \{a,b\} \times \mathbb{R}_+ \to \mathbb{R}_+ \cup \infty$; and a value $V_t^s \in \mathbb{R}_+$, satisfying*

(i) buyers' optimal price posting and search with free entry: for $i \in \{a,b\}$ and $p \in \mathbb{R}_+$,

$$\beta\left[\lambda(\theta_t(i,p))\left(V_{t+1}^m - P_t(i,p)\right) + (1-\lambda(\theta_t(i,p)))\,V_{t+1}^n\right] \le c,$$

with equality if either $p \in \mathbb{P}_t^i$ or $\lambda(\theta_t(a,p)) < 1$.

(ii) sellers' optimal price posting and search: for $i \in \{a,b\}$ and $p \in \mathbb{R}_+$,

$$(1-x)d - c + \beta\left[\gamma(\theta_t(i,p))P_t(i,p) + (1-\gamma(\theta_t(i,p)))\,V_{t+1}^s\right] \le V_t^s,$$

with equality if $\gamma(\theta_t(b,p)) < 1$*, where* V_t^s *is given by*

$$V_t^s = (1-x)d - c + \beta \max_{i \in \{a,b\}, p \in \mathbb{P}_t^i} \left[\gamma(\theta_t(i,p))P_t(i,p) + (1-\gamma(\theta_t(i,p)))V_{t+1}^s\right],$$

or $V_t^s = (1-x)d - c + \beta V_{t+1}^s$ *if* $\mathbb{P}_t^a$ *and* $\mathbb{P}_t^b$ *are both empty.*

Optimal price posting and search ensures that each trader maximizes the value of market participation by either passively directing search or actively seeking to contact a trading partner given the equilibrium relationships between market tightness and posted prices. Free entry into the market drives the present discounted value of market participation for buyers to zero. Parts (i) and (ii) of Definition 1 uphold that perceived market tightness is consistent with a notion of subgame perfection: the function $p \mapsto \theta_t(b,p)$ is such that sellers achieve V_t^s for any bid price including those not advertised in equilibrium, while $p \mapsto \theta_t(a,p)$ is consistent with buyers' indifference about market participation for any possible ask price.[8] The optimal use of the public medium for advertised prices is the novel part of the directed search equilibrium; it requires that buyers and sellers engage in price posting if and only if it is worthwhile to do so.

The following proposition establishes existence of a directed search equilibrium and provides a partial characterization.

Proposition 1. *There exists a directed search equilibrium at time* t*. If* $< \mathbb{P}_t^a, \mathbb{P}_t^b, \theta_t, V_t^s >$ *is an equilibrium, then any* $p \in \mathbb{P}_t^i$ *for* $i \in \{a,b\}$ *yields market tightness* $\theta_t(i,p) = \theta_t^*$ *and expected transaction price* $P_t(i,p) = P_t^*$ *satisfying*

$$c = \beta\left[\lambda(\theta_t^*)\left(V_{t+1}^m - P_t^*\right) + \left(1 - \lambda(\theta_t^*)\right)V_{t+1}^n\right] \tag{11}$$

$$P_t^* = V_{t+1}^s + \eta(\theta_t^*)\left(V_{t+1}^m - V_{t+1}^n - V_{t+1}^s\right) \tag{12}$$

[8]If p is such that a seller cannot achieve V_t^s for any finite buyer-seller ratio, then $\theta_t(b,p) = \infty$. Similarly, $\theta_t(a,p) = 0$ if the expected discounted value of searching among sellers posting ask price p is negative for any positive buyer-seller ratio.

where $\eta(\theta_t^*) = 1 - \theta_t^* \gamma'(\theta_t^*)/\gamma(\theta_t^*)$.

Proposition 1 states that there is an equilibrium market tightness for any price advertised in equilibrium, denoted θ_t^*, and that the expected transaction price is P_t^* in any buyer-seller match. Equation (11) is the free entry condition, and equation (12) designates the equilibrium division of the match surplus (in expectation) between buyer and seller. Proposition 1 does not directly identify the set of prices advertised in equilibrium nor the set of traders posting prices. In some circumstances, ask prices may be useful for directing search. In other instances, bid prices may play a strategic role. Any price advertised in equilibrium must, however, satisfy $P_t(i,p) = P_t^*$. This equality, along with equations (6), (7) and (12), determine $\mathbb{P}_t^a$ and $\mathbb{P}_t^b$. In particular, any $p \in \mathbb{P}_t^a$ must satisfy

$$\min\{0, \Phi_t(p)\} + \int_0^{\max\{0,\Phi_t(p)\}} [1 - F(\phi)]\, d\phi = \eta(\theta_t^*), \tag{13}$$

whereas any $p \in \mathbb{P}_t^b$ must satisfy

$$\max\{0, \Phi_t(p)\} + \int_{\max\{0,\Phi_t(p)\}}^1 [1 - F(\phi)]\, d\phi = \eta(\theta_t^*). \tag{14}$$

The following proposition addresses uniqueness of the equilibrium advertised price and establishes a link between the details of the *ex post* bargaining procedure and the side of the market actively searching in equilibrium.

Proposition 2. *If* $P_t^* \neq \mathrm{E}[\hat{p}_t]$ *or, equivalently,* $\eta(\theta_t^*) \neq \mathrm{E}[\phi]$*,* $\mathbb{P}_t^a \cup \mathbb{P}_t^b$ *is a singleton.* $\mathbb{P}_t^b$ *is empty if* $\eta(\theta_t^*) < \mathrm{E}[\phi]$*, whereas* $\mathbb{P}_t^a$ *is empty if* $\eta(\theta_t^*) > \mathrm{E}[\phi]$.

Sellers advertise ask prices to compete for buyers whenever their expected bargaining strength is too high in the absence of advertised prices. If instead sellers' expected bargaining strength is too low, then buyers advertise bid prices in an effort to compete for sellers. Bargaining fortitude is not necessarily advantageous in a directed search environment: traders prefer to weaken their position by posting a price in an effort to more effectively attract a trading partner. Note that if $P_t^* = \mathrm{E}[\hat{p}_t]$ or, equivalently, $\eta(\theta_t^*) = \mathrm{E}[\phi]$, there are no incentives for buyers or sellers to direct search with advertised prices in equilibrium, although meaningless prices may nonetheless be posted by either side of the market (*e.g.*, a bid price below V_{t+1}^s). Otherwise, search is directed and the equilibrium posted price is unique, given that traders are *ex ante* identical and assets are homogeneous.

Constrained efficiency. To address constrained efficiency, consider a social planner that aims to maximize the welfare of active traders subject to search frictions. The constrained planner designates market tightness, θ_t, and allocates wealth among active traders. Let W_t^n and W_t^s denote the present discounted values associated with buyers' and sellers' participation in the planner's allocation. A constrained efficient allocation maximizes the sum of traders' present discounted values subject to participation constraints and a resource constraint:

$$\max_{\theta_t, W_t^n, W_t^s} W_t^s + \theta_t W_t^n \tag{15}$$

subject to $W_t^n \geq 0$, $W_t^s \geq (1-x)d - c + \beta V_{t+1}^s$, and

$$W_t^s + \theta_t W_t^n \leq (1-x)d - c + \beta V_{t+1}^s - \theta_t \left(c - \beta V_{t+1}^n\right) + \beta\gamma(\theta_t)\left(V_{t+1}^m - V_{t+1}^n - V_{t+1}^s\right). \tag{16}$$

The constrained efficient allocation with $W_t^n = 0$ and $W_t^s = V_t^s$ corresponds to the allocation in a directed search equilibrium.

Proposition 3. *A directed search equilibrium at time t is constrained efficient.*

The constrained efficient level of market activity arises in equilibrium despite the generic inefficiencies of the bargaining procedure and limited commitment to advertised prices. An important feature of the environment for this result is universal access to the price posting technology. When $\eta(\theta_t^*) < \mathrm{E}[\phi]$, sellers advertise prices in equilibrium such that V_t^s is maximized subject to the free entry condition for buyers. If instead $\eta(\theta_t^*) > \mathrm{E}[\phi]$, sellers capture little of the expected surplus in negotiations. Consequently, there would be excessive market entry in the absence of advertised prices relative to a constrained efficient allocation. Sellers sensibly refrain from posting ask prices in these circumstances because such a strategy would only further diminish their expected gains. Advertising a bid price, however, is a suitable strategy from the buyer's perspective in this case because it improves the probability of a match. Bid prices are thus advertised by the demand side of the market and sellers search for buyers in equilibrium such that constrained efficiency is upheld. Advertising capabilities on both sides of the market with search direction determined endogenously are essential for achieving generic constrained efficiency.

Evolution of stocks. The total number of sellers evolves according to

$$s_{t+1} = s_t - \gamma(\theta_t^*)s_t + \delta m_t. \tag{17}$$

Furthermore, since every owner and every seller holds exactly one asset, the total number of sellers and owners must equal the total number of assets in the economy:

$$m_t + s_t = A. \tag{18}$$

Steady state equilibrium. This market has a steady state equilibrium in which all values and prices are determined endogenously and constant over time, and the distribution of traders across states is stationary.

Definition 2. *The steady state equilibrium for this market is a pair of values* (V^m, V^s)*; a market tightness,* θ*; a renegotiation rule,* Φ*; sets of advertised prices,* $\mathbb{P}^a$ *and* $\mathbb{P}^b$*;*

and a distribution of traders across states (m, s, n) *such that:*

(i) values V^m *and* V^s *satisfy*

$$(1-\beta)V^m = d - c - \beta\delta\left[V^m - V^s\right] \tag{19}$$
$$(1-\beta)V^s = (1-x)d - c + \beta\gamma(\theta)\left[1-\eta(\theta)\right]\left[V^m - V^s\right]; \tag{20}$$

(ii) market tightness is the result of free entry: θ *satisfies*

$$c = \beta\lambda(\theta)\left(1-\eta(\theta)\right)\left(V^m - V^s\right); \tag{21}$$

(iii) searching traders follow an optimal renegotiation strategy:

$$\Phi(p) = \frac{p - V^s}{V^m - V^s}; \tag{22}$$

(iv) traders advertise prices strategically: any $p \in \mathbb{P}^a$ *satisfies*

$$\min\{0, \Phi(p)\} + \int_0^{\max\{0,\Phi(p)\}} \left[1 - F(\phi)\right]\, d\phi = \eta(\theta), \tag{23}$$

whereas any $p \in \mathbb{P}^b$ *satisfies*

$$\max\{0, \Phi(p)\} + \int_{\max\{0,\Phi(p)\}}^1 \left[1 - F(\phi)\right]\, d\phi = \eta(\theta); \tag{24}$$

(v) the distribution of traders across states is stationary: (m, s, n) *satisfy*

$$\mathcal{M}(n, s) = \delta m \tag{25}$$
$$s + m = A \tag{26}$$
$$n = \theta s. \tag{27}$$

4 Toronto Standard Taxicab Licenses

Toronto's taxicab licensing system includes, among other classes of licenses, 3,451 *standard taxicab licenses* (STLs). This license type can be used for owner-operated taxicabs, leased to a licensed taxicab driver, rented to shift drivers either directly or through intermediaries, or transferred to a new owner by means of a transaction in a decentralized secondary market. Since a transfer of ownership must be approved and comply with the guidelines of the Municipal Code, every transaction is recorded by the city. This section presents data related to recent STL transactions and advertised prices, and provides evidence that the microstructure of the market corresponds well to the details of the theoretical environment studied in Sections 2 and 3.

4.1 STL Transaction Data

There were 133 STL ownership transfers between September 2013 and August 2014. These include transfers to family members for prices close to zero,[9] as well as many within-family transactions that have prices further from zero but presumably less than market value. In order to focus on market transactions between buyers and sellers that must first search for each other in a decentralized market, STL transfers for amounts less than or equal to 2 are hereinafter excluded from the sample.[10] Transactions between a buyer and a seller that share a common surname are also removed from the sample. Table 1 contains descriptive statistics of the restricted sample of STL transactions.

4.2 STL New Issues Data

It would be of interest to study time on the market for each buyer and seller. Unfortunately, the STL transaction data do not reveal the length of the search process for either trader. Nevertheless, investigation of the database of Business Licence

[9]The City of Toronto's 2012 review of the taxicab industry states that "standard taxicabs cannot be inherited; however, they are often sold to family members for a token amount of 1 dollar."

[10]In the full sample of 133 transactions, there are 6 transactions with a price of 1 and 10 transactions with a price of 2.

Table 1: Summary Statistics of the STL Transaction Data

	obs.	mean	st. error
transaction price	84	177,863	7,023
number of transactions per month	12	7.0000	1.7145

Notes: This table displays summary statistics of the restricted sample of STL transactions from September 2013 to August 2014. Prices are in current CAD.

Renewals and New Issues maintained by Municipal Licensing & Standards provides some clues about the severity of search frictions in the secondary market for STLs. In particular, the time elapsed between an estate transfer and a subsequent market transaction can be recovered from the information recorded in this database. As described in the 2014 report of the taxicab industry prepared by the City of Toronto, the STL is first transferred to the estate of a deceased license holder, but must then be sold to a licensed taxicab driver. The time between these changes in ownership likely reflects (i) the time required to sort out the deceased license holder's estate, (ii) an administrative delay, and (iii) the difficulty in finding a buyer. All three sources of delay are potentially relevant when the STL is sold in the decentralized market, whereas only the first two are important when the STL is transferred to a family member. Comparing the time between transfers for these two groups provides some insight about time on the market for sellers.

Figure 4 plots the empirical distribution functions for the time elapsed between two consecutive changes in STL ownership when the first transfer follows the death of a license holder. Data for Figure 4 include all estate sales recorded between September 2013 and August 2014. The sample is divided into two groups according to market transactions and family transfers.[11] As noted above, there is reason to suspect that

[11] A *family transfer* satisfies at least one of the following two conditions: (i) the recorded price for the transaction is less than or equal to 2, or (ii) the new owner has the same surname as the

longer estate ownership duration for market transactions can be attributed to the time required to advertise or sort through classified ads, contact a potential trading partner, and negotiate the terms of trade. A procedure for deriving an estimate of the average time on the market for sellers from estate ownership duration data is detailed in Appendix B. The results imply that the average time required to find a buyer in the decentralized market is nearly four months.

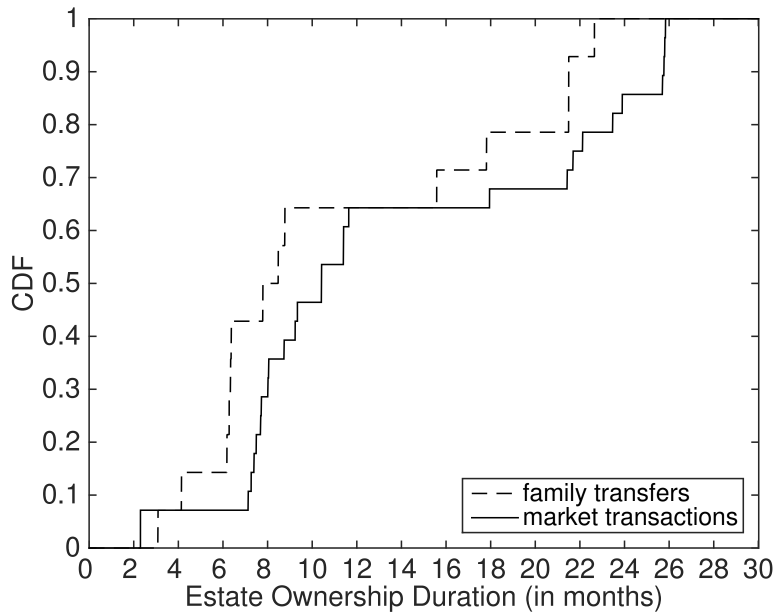

Figure 4: Distribution of Time until New Ownership following Death of STL Owner

4.3 Advertised Price Data

Both sellers and buyers post classified advertisements on `Kijiji.ca`, an online classified service owned by eBay, to convey market participation and attract potential

deceased.

trading partners on the opposite side of the market. Online classified advertisement data were collected from March until August 2014. Each ad is one of two types: either an "I am offering" ad for those offering an item for sale, or an "I want" ad for those on the demand side of the market. Many advertisers on both sides of the market publish a price as part of the ad, although some advertisers omit the price and instead select the option to display "Please Contact" after the "Price" heading. The ad also contains the date listed, a title, a message, and sometimes information about the advertiser including address and phone number. Between March and August 2014, messages posted to `Kijiji.ca` include 57 seller ads and 129 buyer ads. A few of these ads omit prices, but 47 of the 57 ads posted by sellers include ask prices, and 74 of the 129 want ads contain bid prices. Table 2 contains summary statistics of the STL classified advertisement data.

Table 2: Summary Statistics of the STL Classified Advertisement Data

	obs.	mean	st. error
number of seller ads per month	6	9.5000	0.9220
advertised ask price	47	277,285	12,877
number of buyer ads per month	6	21.5000	4.1533
advertised bid price	74	124,473	5,503

Notes: This table displays summary statistics of the STL classified advertisement data from March to August 2014.

For buyers and sellers that advertise their participation in the secondary market for STLs, a link between each classified ad and the corresponding transaction recorded by the city is unfortunately not possible in most cases. Nevertheless, the messages included in the classified advertisements suggest an understanding that posted prices are subject to negotiation. Sellers' ads often include an ask price followed by the phrase "or best offer," or instead provide instructions to "e-mail or call to negotiate." Similarly, advertised bid prices are sometimes accompanied by the qualifier "price is negotiable."

The data presented in this section establish that aspects of the secondary market for STLs align well with features of the theoretical model presented in Section 2; namely, advertised prices, search frictions and price negotiations. Figure 5 displays the distributions of bid prices, ask prices, and actual transaction prices between March and August, 2014. The next section calibrates the model based on the particulars of this market and characterizes the equilibrium of a stochastic version of the parameterized model to demonstrate that the theory can account for advertised and actual price distributions like the ones observed for STLs.

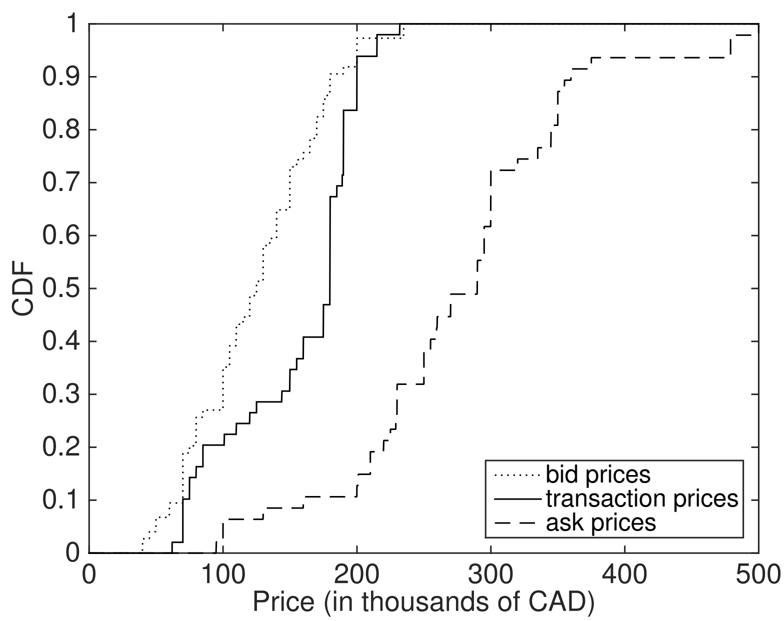

Figure 5: Distributions of Bid Prices, Ask Prices and Transaction Prices

5 Calibration

To view the secondary market for STLs from the perspective of the model developed in Section 2, one can think of the *asset* as a STL and the *dividend*, d, as the profit generated from operating or leasing the STL for a specified period of time.[12] The *separation shock*, δ, reflects retirements, deaths, and/or unanticipated health or financial reasons for wanting/needing to sell the STL when it is infeasible/undesirable to transfer the license to a family member. This Section calibrates the model to the STL market by selecting parameter values to match certain features of the decentralized market for STLs, as well as some of the descriptive statistics reported in Section 4.

The number of assets is set to $A = 3{,}451$, which is the total number of STLs disclosed in the City of Toronto's 2014 report on the taxicab industry. There have been no new STLs issued since 1999. The length of the time period in the model is interpreted as one month. The discount factor, β, is set so that the annual interest rate is 5 percent. The dividend (monthly profit) is set to $d = 1{,}250$, which is based on the average monthly revenue from leasing a STL.[13] In the absence of any guidance for setting the value of the disutility of a mismatch, x is chosen to deliver a value for market tightness in the steady state equal to the ratio of buyers to sellers posting classified ads between March and August 2014 as reported in Table 2.

The matching function is

$$\mathcal{M}(n,s) = \min\left\{n, s, \frac{\alpha n s}{n+s}\right\}, \quad \alpha > 0. \tag{28}$$

[12] Most STL owners are non-drivers that lease the STL to a licensed driver either directly or through an agent. The lessee must operate the taxicab on a regular basis but may also hire part-time shift drivers.

[13] The City of Toronto's 2014 final report on the taxicab industry discloses a current average value for lease revenue per month equal to 1,250. The average monthly lease payment for lessees has been fairly constant in recent years; in December 2011, average monthly lease revenue was 1,244 according to the City of Toronto's 2012 review of the taxicab industry.

Parameters δ, α and c, which are related to separations, search frictions and buyer entry in the model, are chosen to match characteristics of the data described in Section 4. In particular, they determine the average transaction price, the average number of transactions each month, and the expected time on the market for sellers. The first two moments are summary statistics of the STL transaction data from Table 1, while the latter is derived from the estate ownership duration data displayed in Figure 4. More specifically, the average time required to find a buyer in the decentralized market is estimated at 3.8 months. The estimation procedure is detailed in Appendix B. The calibrated parameter values are displayed in Table 3.

Table 3: Calibration of Model Parameters

statistic	value	calibrated parameter	calibrated value
annual interest rate (%)	5	β	0.9959
total number of STLs	3,451	A	3,451
average monthly lease revenue	1,250	d	1,250
average buyer-seller ratio	2.26	x	2.0482
TOM for estate sales (months)	3.8	δ	2.0442×10^{-3}
# of transactions (monthly average)	9.5	α	0.3794
average transaction price	177,863	c	483.6146

Notes: This table reports the model parameter values selected to match certain features of the decentralized secondary market for STLs in Toronto.

Table 4 displays some steady state equilibrium values associated with the calibrated model. The number of sellers is $s = 27$, and the number of traders searching to buy a STL at any point in time is $n = 60$. Replicating the average price and transaction volume in the STL transaction data requires a matching efficiency parameter of 0.38 and a monthly opportunity cost for market participants equal to 484. Search frictions are therefore substantial; it takes on average nearly four months to sell a STL and more than eight months to acquire one in the secondary market.

Table 4: Steady State Results

statistic	calibrated value
number of sellers, s	27
number of buyers, n	60
expected time to buy, $n/\mathcal{M}(n,s)$, in months	8.6
present value of selling, V^s	168,411
present value of owning, V^m	182,039

Notes: This table displays the steady state equilibrium values for the calibrated model.

Note that the distribution F from which the random bargaining parameter, ϕ, is drawn was not specified in the calibration exercise. The parameters of the model were instead selected according to Proposition 1 to match the average transaction price. In other words, one can remain agnostic about the precise details of the *ex post* bargaining procedure by applying the steady state version of equation (12) under the assumption that traders advertise bid/ask prices strategically and adhere to optimal renegotiation strategies. What follows is a stochastic version of the calibrated model with a particular distributional assumption for F to uncover equilibrium prices advertised by buyers and sellers according to equations (13) and (14).

5.1 Dynamic Stochastic Equilibrium

Consider the following distribution function for the random bargaining parameter:

$$F(\phi) = \begin{cases} 1 - \pi & \text{if } 0 \leq \phi < 1 \\ 1 & \text{if } \phi = 1 \end{cases} \tag{29}$$

The *ex post* bargaining game is therefore a random dictator mechanism; the seller (buyer) is randomly selected to make a take-it-or-leave-it offer with probability π

$(1-\pi)$. The value of π is chosen so that advertised prices are irrelevant in the steady state, which in this case requires

$$\pi = \mathrm{E}[\phi] = \eta(\theta) = \frac{P - V^s}{V^m - V^s}, \tag{30}$$

where the absence of a time subscript denotes a steady state value. To generate advertised prices out of the steady state, I introduce random perturbations to the monthly cost of market participation to simulate variation in the value of other potential investment opportunities.More specifically, let the opportunity cost of market participation evolve according to

$$c_t = (1-\rho)c + \rho c_{t-1} + \varepsilon_t, \tag{31}$$

where ε_t is an i.i.d. normal random variable with mean 0 and standard deviation σ_ε. Changes in the monthly cost of market participation influence the buyer-seller ratio via the free entry condition. As market tightness fluctuates around its steady state value, advertised bid and ask prices become important for directing search.

Computing the equilibrium of the dynamic stochastic model generates the transaction price distribution and advertised price distributions plotted in Figure 6.[14] These simulated price distributions mirror, to some extent, the empirical distribution functions for advertised and actual prices for STLs plotted in Figure 5; advertised bid prices tend to be less than transaction prices, while advertised ask prices tend to exceed transaction prices. The qualitative similarities between Figures 5 and 6 suggest that advertised prices in the secondary market for STLs affect the incidence of price negotiations and play a role in directing search in the manner proposed in the

[14]For this illustration, parameters ρ and σ_ε for the stochastic process are, for no particularly good reason, set to 0.95 and 30.

theory.[15] More generally, the model's ability to replicate the aforementioned characteristics of the STL price distributions is predicated on two important features of the theoretical framework: (i) advertised prices on both sides of the market that play a role in directing search, and (ii) negotiated increases (discounts) from advertised bid (ask) prices.

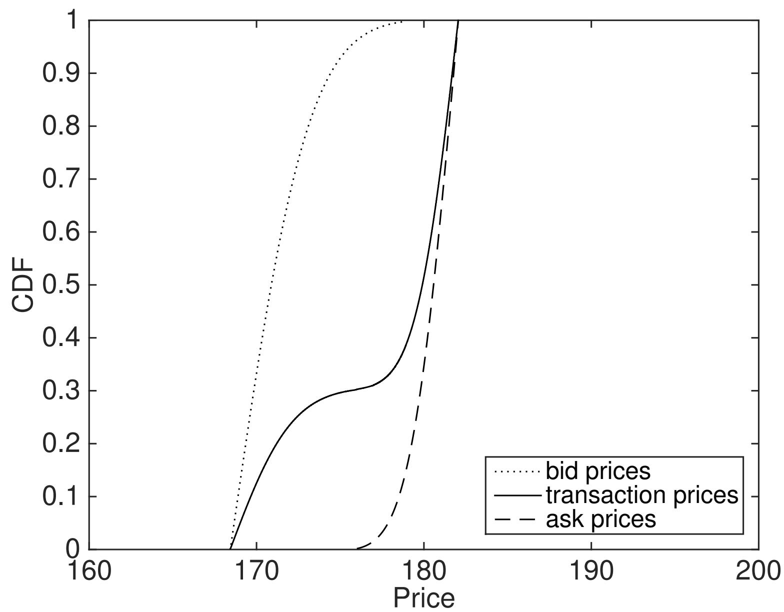

Figure 6: Simulated Distributions of Bid Prices, Ask Prices and Transaction Prices

[15]There is no attempt here to account for the high degree of price dispersion in the data owing to the limited heterogeneity in the model. Despite the fact that STLs are essentially identical assets, it could be that buyers and sellers in the secondary market for STLs are heterogeneous in terms of their preferences (*e.g.*, traders may differ in terms of their patience, β, or the severity of their separation shock, x). There could also be productivity differences among taxicab drivers, so that the dividend, d, is owner-specific or license-specific depending on whether the STL is owner-operated or leased. In any case, it is likely that whatever is absent from the model that would generate the observed dispersion in transaction prices (*e.g.*, trader heterogeneity or idiosyncratic fluctuations in the stream of dividends) would also account for the dispersion in advertised prices. Furthermore, since it is essentially costless to post an ad, traders that actively search may themselves post meaningless prices (*e.g.*, an ask price exceeding V^m_{t+1} or a bid price below V^s_{t+1}), which would contribute to the observed dispersion in posted prices.

It is worth remarking that these relationships between advertised and actual price distributions differ from those predicted by existing directed search models that adopt a reserve price interpretation of a posted price. For example, one would expect prices advertised by sellers to be lower than transaction prices when there is *ex post* competition among buyers (*e.g.*, Peters and Severinov (1997) and Julien, Kennes, and King (2000)). Similarly, a directed search model that features pre-match price posting by buyers and post-match price demands by competing sellers would imply transaction prices in equilibrium below those advertised by the demand side of the market. It is sometimes assumed that buyers first advertise initial offers and subsequently increase their bids if other buyers contact the same seller (*e.g.*, Albrecht, Gautier, and Vroman (2006)). Consequently, transaction prices exceed the offers announced initially by buyers, but by assumption there are no advertised prices on the supply side of the market. Although these models predict transaction prices that sometimes differ from posted prices, the implied relationships between price distributions do not resemble those in the market for STLs. In this paper, accounting for price distributions like those in Figure 5 requires the opportunity for both sides of the market to advertise meaningful prices that are subject to negotiation.

6 Conclusion

In this paper I develop a theoretical model of a decentralized asset market with trading frictions. The trading process has three important features: pre-match communication, search frictions, and a strategic method of price determination. Traders on both sides of the market have the opportunity to post a public advertisement containing a bid or ask price. Traders that do not advertise a price instead observe all price announcements and search for a trading partner by targeting a particular advertised price. Transaction prices are then determined in a bilateral bargaining game between a matched buyer and seller, where the advertised price is interpreted as the initial offer in an alternating offer bargaining game. The advertisement therefore conveys commitment to a bid or ask price, but the counterparty that has not engaged in pre-match price publication maintains the ability to trigger *ex post* negotiation. In a setting where the outcome of negotiations is not entirely predictable, the advertised price provides incentive for potential trading partners to direct their search. The possibility of unfavorable outcomes in the bargaining procedure therefore supports the commonplace strategy of posting bid or ask prices even when advertised prices tend to be different from transaction prices.

The decentralized secondary market for standard taxicab licenses (STLs) exhibits many of the features modeled in Section 2; there is evidence of search frictions, pre-match price announcements and *ex post* bargaining, as discussed in Section 4. Moreover, the assets being traded are essentially identical, which rules out alternative interpretations of advertised prices that rely on idiosyncratic values and costly inspection.[16] This application is therefore well-suited for examining the link between advertised prices and transaction prices in a relatively inactive decentralized asset market with very limited heterogeneity in the asset. A stochastic version of the calibrated model produces distributions of advertised and transaction prices that have qualitative properties in common with the empirical distributions given by the STL data. These similarities depend critically on two novel features of the model: (i) *ex ante* uncertainty about the exact details of the *ex post* bargaining procedure, and (ii) strategic price posting decisions by traders on both sides of the market.

[16] For example, Chen and Rosenthal (1996a,b) and Arnold (1999) propose that asking prices provide incentives for buyers to incur an inspection cost to learn their idiosyncratic valuation of the item for sale. The asking price, which represents commitment to a price ceiling, guarantees additional surplus to the buyer whenever the inspection reveals to both parties that the buyer's willingness to pay is high.

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A Omitted Proofs

Proof of Proposition 1

Adopting a similar structure to that found in Acemoglu and Shimer (1999), we prove Proposition 1 in three steps. In step 1 we establish that if $< \mathbb{P}_t^a, \mathbb{P}_t^b, \theta_t, V_t^s >$ is a directed search equilibrium at time t, then any $p \in \mathbb{P}_t^i$ for $i \in \{a, b\}$ is such that $P_t^* = P_t(i, p)$ and $\theta_t^* = \theta_t(i, p)$ solve

$$\max_{P,\theta} \gamma(\theta) \left(P - V_{t+1}^s\right) \tag{32}$$

subject to

$$\beta \left[\lambda(\theta) \left(V_{t+1}^m - P\right) + (1 - \lambda(\theta)) V_{t+1}^n\right] = c. \tag{33}$$

In step 2 we prove the converse: if some (P_t^*, θ_t^*) solves this problem and $P_t(i, p) = P_t^*$, then there exists an equilibrium with $p \in \mathbb{P}_t^i$ and $\theta_t(i, p) = \theta_t^*$. In step 3 we show that there exists a unique solution to this problem, denoted (P_t^*, θ_t^*), and that there exist $i \in \{a, b\}$ and $p \in \mathbb{R}_+$ such that $P_t(i, p) = P_t^*$.

Step 1. Let $< \mathbb{P}_t^a, \mathbb{P}_t^b, \theta_t, V_t^s >$ be a directed search equilibrium at time t with $p \in \mathbb{P}_t^i$ for $i \in \{a, b\}$. Let $P_t^* = P_t(i, p)$ and $\theta_t^* = \theta_t(i, p)$. Part (i) of Definition 1 and $p \in \mathbb{P}_t^i$ guarantee that (P_t^*, θ_t^*) satisfies constraint (33). Part (ii) of Definition 1 and $p \in \mathbb{P}_t^i$ imply

$$(1 - x)d - c + \beta \left[\gamma(\theta_t^*) P_t^* + (1 - \gamma(\theta_t^*)) V_{t+1}^s\right] = V_t^s.$$

Now consider ask price V_{t+1}^s so that $P_t(a, V_{t+1}^s) = V_{t+1}^s$. Part (ii) of Definition 1 then implies

$$(1 - x)d - c + \beta V_{t+1}^s \leq V_t^s. \tag{34}$$

Suppose another pair (P', θ') achieves a higher value of the objective so that

$$(1-x)d - c + \beta\left[\gamma(\theta')P' + (1-\gamma(\theta'))V_{t+1}^s\right] > V_t^s. \tag{35}$$

Combining (34) and (35) establishes that $P' > V_{t+1}^s$. To establish that (P', θ') cannot satisfy constraint (33), suppose (FSOC) that it does:

$$\beta\left[\lambda(\theta')\left(V_{t+1}^m - P'\right) + (1-\lambda(\theta'))\,V_{t+1}^n\right] = c$$

Given that $\beta V_{t+1}^n < c$, this requires $P' < V_{t+1}^m - V_{t+1}^n$.

By part (iv) of Lemma 3 there exists a $p' \in \mathbb{R}_+$ and $i' \in \{a, b\}$ such that $P_t(i', p') = P'$. Since $< \mathbb{P}_t^a, \mathbb{P}_t^b, \theta_t, V_t^s >$ is a directed search equilibrium, part (ii) of Definition 1 requires

$$(1-x)d - c + \beta\big[\gamma(\theta_t(i', p'))P_t(i', p') + (1-\gamma(\theta_t(i', p')))V_{t+1}^s\big] \leq V_t^s. \tag{36}$$

Inequalities (35) and (36) and $P' > V_{t+1}^s$ imply $\gamma(\theta') > \gamma(\theta_t(i', p'))$, and hence $\lambda(\theta') < \lambda(\theta_t(i', p'))$. Therefore,

$$\beta\lambda(\theta')\left(V_{t+1}^m - V_{t+1}^n - P'\right) < \beta\lambda(\theta_t(i', p'))\left(V_{t+1}^m - V_{t+1}^n - P_t(i', p')\right) \leq c - \beta V_{t+1}^n,$$

where the first inequality uses $\lambda(\theta_t(i', p')) > \lambda(\theta')$ and $P_t(i', p') = P' < V_{t+1}^m - V_{t+1}^n$, and the last inequality applies part (i) of Definition 1. This contradiction establishes that the pair (P', θ') must violate constraint (33).

Step 2. Let (P_t^*, θ_t^*) denote a solution to the constrained optimization problem and consider a pair (i^*, p^*) satisfying $P_t(i^*, p^*) = P_t^*$. Construct an equilibrium as follows: $\mathbb{P}_t^{i^*} = \{p^*\}$; $\mathbb{P}_t^{-i^*} = \emptyset$;

$$V_t^s = (1-x)d - c + \beta\left[\gamma(\theta_t^*)P_t^* + (1-\gamma(\theta_t^*))V_{t+1}^s\right];$$

and let θ_t satisfy

$$\beta\left[\lambda(\theta_t(a,p))\left(V_{t+1}^m - P_t(a,p)\right) + (1-\lambda(\theta_t(a,p)))\,V_{t+1}^n\right] = c,$$

or $\theta_t(a,p) = 0$ if there is no solution to this equation, and

$$(1-x)d - c + \beta\left[\gamma(\theta_t(b,p))P_t(b,p) + (1-\gamma(\theta_t(b,p)))V_{t+1}^s\right] = V_t^s,$$

or $\theta_t(b,p) = \infty$ if there is no solution to this equation.

Notice that since the pair $(P,\theta) = (V_{t+1}^m - c/\beta, 0)$ satisfies constraint (33), it must be that

$$\gamma(\theta_t^*)(P_t^* - V_{t+1}^s) \geq 0$$

which implies $P_t^* \geq V_{t+1}^s$ and $V_t^s \geq (1-x)d - c + \beta V_{t+1}^s$.

It is clear that $< \mathbb{P}_t^a, \mathbb{P}_t^b, \theta_t, V_t^s >$ satisfies part (i) of Definition 1 with $i = a$ and part (ii) with $i = b$.

Suppose (FSOC) that some pair (b, p') violates part (i) of Definition 1:

$$\beta\left[\lambda(\theta_t(b,p'))\left(V_{t+1}^m - P_t(b,p')\right) + (1-\lambda(\theta_t(b,p')))\,V_{t+1}^n\right] > c.$$

Given that $\beta V_{t+1}^n < c$, it follows that $\theta_t(b,p') < \infty$. Choose $\theta' > \theta_t(b,p')$ satisfying

$$\beta\left[\lambda(\theta')\left(V_{t+1}^m - P_t(b,p')\right) + (1-\lambda(\theta'))\,V_{t+1}^n\right] = c.$$

By the construction of θ_t, $\theta' > \theta_t(b, p')$ and $P_t(b, p') \geq V^s_{t+1}$ (see part (iii) of Lemma 3) imply

$$(1 - x)d - c + \beta \left[\gamma(\theta')P_t(b, p') + (1 - \gamma(\theta'))V^s_{t+1}\right] > V^s_t.$$

The pair $(P_t(b, p'), \theta')$ satisfies constraint (33) and achieves a higher value of the objective than (P^*_t, θ^*_t): a contradiction.

Suppose (FSOC) that some pair (a, p'') violates part (ii) of Definition 1:

$$(1 - x)d - c + \beta \left[\gamma(\theta_t(a, p''))P_t(a, p'') + (1 - \gamma(\theta_t(a, p'')))V^s_{t+1}\right] > V^s_t.$$

Since it was previously established that $V^s_t \geq (1-x)d-c+\beta V^s_{t+1}$, this strict inequality requires $\theta_t(a, p'') > 0$. Then, by the construction of θ_t,

$$\beta \left[\lambda(\theta_t(a, p'')) \left(V^m_{t+1} - P_t(a, p'')\right) + (1 - \lambda(\theta_t(a, p''))) V^n_{t+1}\right] = c.$$

The pair $(P_t(a, p''), \theta_t(a, p''))$ satisfies constraint (33) and achieves a higher value of the objective than (P^*_t, θ^*_t): a contradiction.

Step 3. Substituting constraint (33) into objective (32) yields the following optimization problem:

$$\max_{\theta} \beta\gamma(\theta) \left(V^m_{t+1} - V^n_{t+1} - V^s_{t+1}\right) - \theta \left(c - \beta V^n_{t+1}\right) \tag{37}$$

The objective in (37) is continuous and concave in θ. Let $\bar{\theta} \in (0, \infty)$ denote the unique solution to $\beta \left[\lambda(\bar{\theta}) \left(V^m_{t+1} - V^s_{t+1}\right) + \left(1 - \lambda(\bar{\theta})\right) V^n_{t+1}\right] = c$. The objective in (37) is zero when evaluated at $\theta = 0$ and $\theta = \bar{\theta}$. These properties ensure that the solution to problem (37) is unique and on the interior of $[0, \bar{\theta}]$. Let θ^*_t denote the solution and let P_t^* satisfy constraint (33) with $\theta = \theta_t^*$:

$$\beta \left[\lambda(\theta_t^*) \left(V_{t+1}^m - P_t^*\right) + (1 - \lambda(\theta_t^*))\, V_{t+1}^n\right] = c. \tag{38}$$

The pair (P_t^*, θ_t^*) represents the unique solution to the constrained optimization problem. By part (iv) of Lemma 3, there exist i^* and p^* such that $P_t(i^*, p^*) = P_t^*$. By step 2, the pairs (i^*, p^*) and (P_t^*, θ_t^*) can be used to construct an equilibrium. Moreover, the result established in step 1 ensures that any $p \in \mathbb{P}_t^i$ for $i \in \{a, b\}$ is such that $P_t(i, p) = P_t^*$ and $\theta_t(i, p) = \theta_t^*$

To show that (P_t^*, θ_t^*) satisfies (12), we derive the following first order condition, which is both necessary and sufficient to identify θ_t^*:

$$\beta \left[\gamma'(\theta_t^*) \left(V_{t+1}^m - V_{t+1}^s\right) + (1 - \gamma'(\theta_t^*))\, V_{t+1}^n\right] = c. \tag{39}$$

Combining (38), (39) and $\gamma(\theta_t^*) = \theta_t^* \lambda(\theta_t^*)$, and rearranging for P_t^* yields

$$P_t^* = V_{t+1}^s + \left(1 - \frac{\theta_t^* \gamma'(\theta_t^*)}{\gamma(\theta_t^*)}\right) \left(V_{t+1}^m - V_{t+1}^n - V_{t+1}^s\right). \quad \square \tag{40}$$

Proof of Proposition 2

It was established in step 3 of the proof of Proposition 1 that θ_t^* is unique, positive and finite. Since η is the elasticity of the matching function with respect to s, Assumption 1 and the properties of $\mathcal{M}$ imply $0 < \eta(\theta_t^*) < 1$. First consider the case in which $0 < \eta(\theta_t^*) < \mathrm{E}[\phi]$. Given (3) and (12), these inequalities are equivalent to $V_{t+1}^s < P_t^* < \mathrm{E}[\hat{p}_t]$. Parts (i) and (ii) of Lemma 3 imply a unique (a, p) satisfying $P_t(a, p) = P_t^*$, while part (iii) of Lemma 3 precludes a solution to $P_t(b, p) = P_t^*$. $\mathbb{P}_t^a$ is therefore a singleton and $\mathbb{P}_t^b = \emptyset$.

Next consider $\mathrm{E}[\phi] < \eta(\theta_t^*) < 1$ which, given (3) and (12), is equivalent to $\mathrm{E}[\hat{p}_t] < P_t^* < V_{t+1}^m - V_{t+1}^n$. Parts (i) and (iii) of Lemma 3 imply a unique (b,p) satisfying $P_t(b,p) = P_t^*$, while part (ii) of Lemma 3 precludes $P_t(a,p) = P_t^*$. In this case, $\mathbb{P}_t^b$ is a singleton and $\mathbb{P}_t^a = \emptyset$. □

Proof of Proposition 3

The resource constraint (16) binds and the constrained social planner's problem can be written

$$\max_{\theta_t, W_t^n, W_t^s} \beta\gamma(\theta_t)\left(V_{t+1}^m - V_{t+1}^n - V_{t+1}^s\right) - \theta_t\left(c - \beta V_{t+1}^n\right) \tag{41}$$

subject to $W_t^n \geq 0$ and $W_t^s \geq (1-x)d - c + \beta V_{t+1}^s$. Problem (41) is the same as problem (37) with the additional possibility of transferring wealth among traders subject to participation constraints. Given that W_t^n and W_t^s do not appear in the objective function, they do not affect the planner's choice of θ_t. Therefore, the triple $(\theta_t, W_t^n, W_t^s) = (\theta_t^*, 0, V_t^s)$ solves the planner's problem and the equilibrium is constrained efficient. □

B Estimating Time on the Market from Estate Ownership Duration

Suppose there is an administrative delay of τ periods associated with the official transfer of ownership of a STL, and that sorting out a deceased STL holder's estate is a process that terminates each period with probability μ. Afterwards, absent a family transfer, a suitable buyer is found and ownership of the STL is transferred from the estate to the buyer by means of a transaction in the decentralize market each period with probability $\gamma(\theta)$. Let d^j denote the duration of estate ownership (*i.e.*, time elapsed between transfer to and from the estate of the deceased STL owner) for an STL that is ultimately transferred to a family member ($j = F$) or sold in the decentralized market ($j = M$). Duration d^F is therefore a geometrically distributed random variable with CDF

$$\text{Prob}\left\{d^F \leq d\right\} = \mu \sum_{k=0}^{d-\tau-1} (1-\mu)^k = 1 - (1-\mu)^{d-\tau}, \tag{42}$$

if $d > \tau$ and $\text{Prob}\left\{d^F \leq d\right\} = 0$ otherwise. Duration d^M, on the other hand, is a random variable with CDF

$$\begin{aligned} \text{Prob}\left\{d^M \leq d\right\} &= \mu\gamma(\theta) \sum_{k=0}^{d-\tau-2} \sum_{h=0}^{d-\tau-2-k} (1-\mu)^k (1-\gamma(\theta))^h \\ &= 1 - \frac{\mu(1-\gamma(\theta))^{d-\tau} - \gamma(\theta)(1-\mu)^{d-\tau}}{\mu - \gamma(\theta)}. \end{aligned} \tag{43}$$

if $d > \tau + 1$ and $\text{Prob}\left\{d^M \leq d\right\} = 0$ otherwise. The administrative delay, τ, and probabilities μ and $\gamma(\theta)$ can then be estimated from the estate ownership duration data by maximizing the following log-likelihood function:

$$\begin{aligned} \log \mathcal{L}(\mu, \tau, \gamma) = &\sum_{d \in \mathbb{D}^F} \log\left(\mu(1-\mu)^{d-\tau-1}\right) \\ &+ \sum_{d \in \mathbb{D}^M} \log\left(\frac{\mu\gamma}{\mu-\gamma}\left[(1-\gamma)^{d-\tau-1} - (1-\mu)^{d-\tau-1}\right]\right) \end{aligned} \tag{44}$$

where $\mathbb{D}^F$ represents the set of estate ownership durations recorded for family transfers, and $\mathbb{D}^M$ is the same for market transactions. With ownership duration (measured in days) for all estate sales recorded between September 2013 and August 2014, this procedure yields parameter estimates $\hat{\mu}_{\text{seq}} = 3.8455 \times 10^{-3}$, $\hat{\tau}_{\text{seq}} = 52$ and $\hat{\gamma}_{\text{seq}} = 8.5738 \times 10^{-3}$. The corresponding distribution functions for time until new ownership for market transactions and family transfers are displayed in Figure 7. The implication for expected time on the market for sellers (*i.e.*, average time spent searching/waiting for a buyer) is therefore $1/\hat{\gamma}_{\text{seq}} = 117$ days, or approximately 3.8 months.

Differences after the first 12 months might be particularly relevant because extending estate ownership beyond one year requires periodic approval by the Toronto Licensing Tribunal, implying that unnecessary delay is no longer costless.[17] To repeat the estimation procedure using only estate ownership duration beyond the first 365 days, it is necessary to increase the sample size by, for example, considering all estate sales recorded between September 2011 and August 2014. The parameter estimates in this case are $\hat{\mu}_{>365} = 6.0197 \times 10^{-3}$ and $\hat{\gamma}_{>365} = 7.9745 \times 10^{-3}$. As discussed above, $\hat{\mu}_{\text{seq}} < \hat{\mu}_{>365}$ likely reflects the absence of pressure from the Toronto Licensing Tribunal to expedite family transfers the STL until one year after the STL is issued to the estate. Figure 8 displays the CDF for these parameter estimates, and the expected time on the market for sellers is 125 days. The two approaches yield quite similar implications for time on the market.

This procedure may in fact understate trading frictions in the decentralized market by assuming that the executorial period and trading delay occur sequentially. It is entirely possible that an STL is advertised for sale before other non-search-related sources of delay are resolved by the representative of a deceased STL owner. As a useful benchmark for comparison, consider an alternative assumption that both the sorting of the estate and the search process occur simultaneously. A market transaction occurs only after both activities have terminated. In this setting, d^M is a

[17] In contrast, there could be reasons to delay the transfer during the first 12 months if the intended beneficiary is not yet a licensed taxicab driver, or if the revenue generated from leasing the STL in the interim can be allocated to someone else.

random variable with CDF

$$\begin{aligned}\text{Prob}\left\{d^M \le d\right\} &= \left(\mu \sum_{k=0}^{d-\tau-1} (1-\mu)^k\right)\left(\gamma(\theta) \sum_{k=0}^{d-\tau-1} (1-\gamma(\theta))^k\right) \\ &= \left[1-(1-\mu)^{d-\tau}\right]\left[1-(1-\gamma(\theta))^{d-\tau}\right]. \end{aligned} \tag{45}$$

if $d > \tau$ and $\text{Prob}\left\{d^M \le d\right\} = 0$ otherwise. The appropriate log-likelihood function to be maximized is

$$\begin{aligned}\log \mathcal{L}(\mu,\gamma,\tau) = &\sum_{d\in\mathbb{D}^F} \log\left(\mu(1-\mu)^{d-\tau-1}\right) \\ &+ \sum_{d\in\mathbb{D}^M} \log\left(\begin{array}{l} \mu(1-\mu)^{d-\tau-1}\left[1-(1-\gamma)^{d-\tau}\right] \\ \quad +\gamma(1-\gamma)^{d-\tau-1}\left[1-(1-\mu)^{d-\tau-1}\right]\end{array}\right)\end{aligned} \tag{46}$$

This procedure with simultaneous delays yields parameter estimates $\hat{\mu}_{\text{sim}} = 3.7047 \times 10^{-3}$, $\hat{\tau}_{\text{sim}} = 53$ and $\hat{\gamma}_{\text{sim}} = 4.4736 \times 10^{-3}$. The CDFs (not shown) look almost indistinguishable from those in Figure 7, however the implied average time on the market for sellers is $1/\hat{\gamma}_{\text{sim}} = 224$ days, or approximately 7.3 months, which is substantially longer than under the sequential delay assumption.

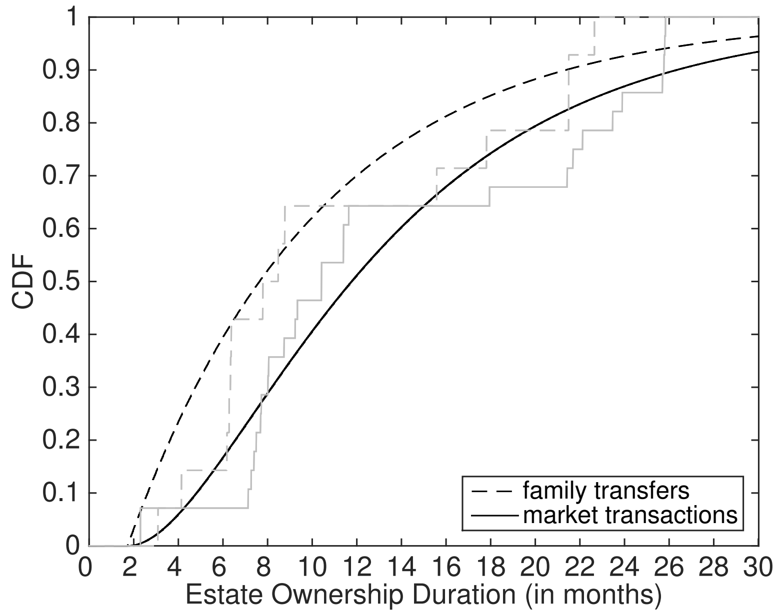

Figure 7: Estimated CDF of Time until New Ownership following Death of STL Owner

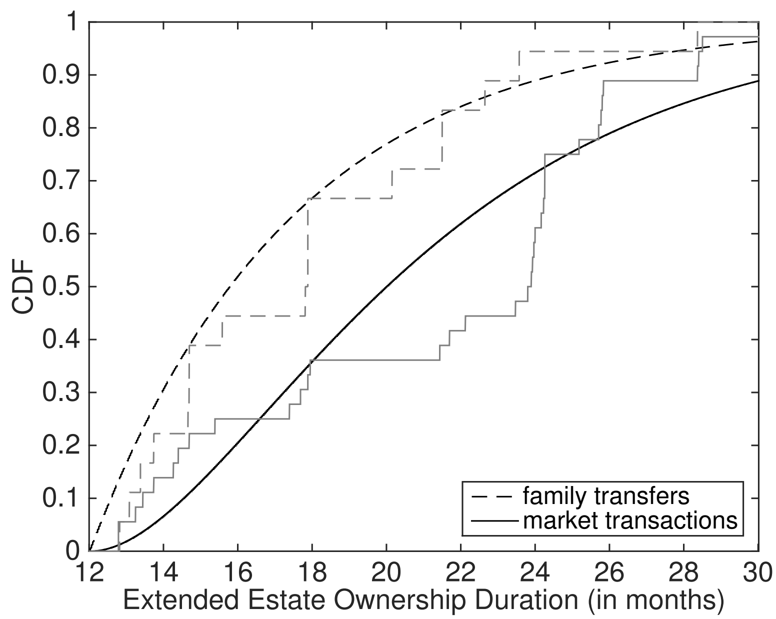

Figure 8: Estimated CDF of Time until New Ownership Exceeding 12 Months following Death of STL Owner